FTUV–01-0505 IFIC–01-24
May 5, 2001

Superalgebra cohomology, the geometry of extended superspaces and superbranes

J. A. de Azcárraga[1] and J. M. Izquierdo[2] [1]

1 *Departamento de Física Teórica, Universidad de Valencia and IFIC, Centro Mixto Universidad de Valencia–CSIC, E–46100 Burjassot (Valencia), Spain*

2 *Departamento de Física Teórica, Universidad de Valladolid E–47011 Valladolid, Spain*

Abstract

We present here a cohomological analysis of the new spacetime superalgebras that arise in the context of superbrane theory. They lead to enlarged superspaces that allow us to write D-brane actions in terms of fields associated with the additional superspace variables. This suggests that there is an extended superspace/worldvolume fields democracy for superbranes.

[1]Invited lecture delivered at the XXXVII Karpacz Winter School on *New Developments in Fundamental Interaction Theories*, February 6-15, 2001, Karpacz, Poland. To be published in Proceedings.
e-mail addresses: j.a.de.azcarraga@ific.uv.es, izquierd@fta.uva.es

1 Introduction

Due to the development of the new string theory, it has become clear that the supersymmetry algebra contains new bosonic tensorial generators, which are central if one does not consider the Lorentz part of the complete algebra. The study of the structure of the supersymmetry algebra goes back to [1], and tensorial charges were already considered in [2] (see also [3, 4]). An enlarged superalgebra with an additional 'central' fermionic generator was introduced in [5]; other, more general algebras were considered in [6], where it was proved that to every super-p-brane of the branescan in [7], corresponds a new spacetime superalgebra, generalizing the results of [5, 8]. The point of view in [6, 9] constitutes the Lie superalgebra counterpart of the Chevalley-Eilenberg (CE) supersymmetry algebra cohomology (see [10, 11]) analysis of the scalar branes previously done in [12]. We report here on a recent work [13] which leads to a systematic cohomological construction of all these algebras, with both bosonic and fermionic generators, as well as their associated extended superspace $\tilde{\Sigma}$ *groups*. These contain additional coordinates, besides those (x^μ, θ^α) of the standard superspace Σ, which can be used to construct manifestly invariant super-p-brane Wess-Zumino (WZ) terms [8, 6, 9, 13]. Indeed, it is well known (see *e.g.* [10]) that the quasi-invariance of Lagrangians (invariance but for a total derivative) exhibits the non-trivial cohomology of the symmetry group, and that they can be rendered manifestly invariant by using the additional variables associated with the extended group. This is reflected in the realization of the symmetries in terms of Noether currents and charges, and we shall provide their general expression for superbranes. As one might expect, the new variables on $\tilde{\Sigma}$ appear trivially (in total derivatives) in the action of the scalar super-p-branes. In the case of D-branes, however, some new variables appear non-trivially since the worldvolume fields can be constructed as pull-backs of suitably enlarged superspaces that correspond to new superalgebras with additional fermionic generators.

2 Extended superspaces given by central extensions of the supertranslation group

Standard superspace itself provides the simplest example of our point of view. Consider the abelian, odd, *supertranslation group* sTr_D, of group law $\theta''^\alpha = \theta'^\alpha + \theta^\alpha$ (generically, we denote a group law as $g'' = g'g \equiv L_{g'}g = R_g g'$; L and R are the left and right actions of the group on itself). Associated with sTr_D is the trivial Maurer-Cartan (MC) equation $d\Pi^\alpha = 0$ $(\Pi^\alpha = d\theta^\alpha)$, which is the dual version of the corresponding abelian Lie superalgebra $\{D_\alpha, D_\beta\}$=0 [1]. Now, let θ^α be Majorana. Then, $(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha \wedge \Pi^\beta$ defines a *Tr_D-valued non-trivial CE two-cocycle* since a) it is closed and left-invariant (LI) *i.e.*, it is a CE cocycle and b) it is not d of a LI form (not a coboundary). Therefore, it is consistent to extend $d\Pi^\alpha = 0$ by a one-form $\tilde{\Pi}^\mu$ $(\tilde{\Pi}^\mu \equiv (1/2)(C\Gamma^\mu)_{\alpha\beta}\theta^\alpha d\theta^\beta$, say) so that

$$d\tilde{\Pi}^\mu = (1/2)(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta \quad . \tag{1}$$

Clearly, $\tilde{\Pi}^\mu$ and Π^α define a free differential algebra (FDA) but they are not the MC one-forms of a *Lie* algebra since $\tilde{\Pi}^\mu$ *is not LI.* To remedy this, we introduce a *new group coordinate* x^μ *– Minkowski space Tr_D–* and define instead

$$\Pi^\mu = dx^\mu + \tilde{\Pi}^\mu = dx^\mu + (1/2)(C\Gamma^\mu)_{\alpha\beta}\theta^\alpha d\theta^\beta \quad , \quad \mu = 0, 1, \ldots, D-1 \quad . \tag{2}$$

Obviously, $d\Pi^\mu = d\tilde{\Pi}^\mu$ and we may now choose the transformation law for x^μ so that Π^μ is LI,

$$x''^\mu = x'^\mu + x^\mu - (1/2)(C\Gamma^\mu)_{\alpha\beta}\theta'^\alpha\theta^\beta \quad . \tag{3}$$

This gives [14, 10] *rigid superspace* Σ as a *group* parametrized by (θ^α, x^μ), with group law now given by $\theta''^\alpha = \theta'^\alpha + \theta^\alpha$ and (3): *supersymmetry is the result of the non-trivial cohomology of the odd supertranslation group.*

The philosophy behind this simple superspace example, that 'fermions (θ's) are first' and that *rigid superspaces are group extensions*, may be extended by considering other types of two-cocycles (*i.e,*

[1] We use D_α (covariant derivatives) rather than Q_α (supersymmetry generators) because we deal with LI (hence, supersymmetry invariant) forms and vector fields, but this is unessential: the left and right algebras have the same structure constants but for an overall sign that may be conveniently ignored here.

valued on more general spaces than Tr_D) on the sTr_D algebra. Explicitly, given a particular sTr_D algebra to be extended, one

a) looks for a non-trivial CE two-cocycle of the desired Lorentz-covariant nature. This means searching for Lorentz-tensor-valued LI closed two-forms that are not d(LI one-form).

b) Introduces a new LI one-form, the differential of which is the two-cocycle. Then,

c) the left invariance of the new one-form is achieved by fixing the transformation properties of the *new group coordinate*. This defines in general an *extended superspace (super)group* manifold $\tilde{\Sigma}$.

d) The new LI one-form together with the MC equations automatically define by (LI one-forms/LI vector fields) duality an extended Lie algebra.

e) Since the required Lorentz group symmetry is implicit in the process, the extension cocycles must be covariant under the action of $Spin(1, D-1)$.

The extension procedure described above can be applied more than once.

Consider the *general case of 'central' bosonic extensions* of sTr_D (they are really tensorial, since the Lorentz generators do not commute with the 'central' ones). As in the above superspace example, we may look at the problem from the Lie algebra $\mathcal{G}$ or the group G point of view:

1) *Lie algebra extension point of view*
When described in terms of LI forms, the algebra extensions require the existence of higher order (α, β)-*symmetric* Lorentz tensors $(C\Gamma^{\mu_1 \cdots \mu_p})_{\alpha\beta}$ of rank p

$$d\Pi^{\mu_1 \cdots \mu_p} \equiv (1/2)(C\Gamma^{\mu_1 \cdots \mu_p})_{\alpha\beta} \Pi^\alpha \Pi^\beta \tag{4}$$

($\Pi^\alpha \Pi^\beta \equiv \Pi^\alpha \wedge \Pi^\beta = \Pi^\beta \wedge \Pi^\alpha$; we omit wedge products). The corresponding generators $Z_{\mu_1 \ldots \mu_p}$ are all ($D_\alpha-$)*central*, as the translation generator $X_\mu = P_\mu$ itself, and are associated with new central charges.

The LI of the new forms in (4) *requires* new group parameters $\varphi^{\mu_1 \cdots \mu_p}$ so that

$$\Pi^{\mu_1 \cdots \mu_p} = d\varphi^{\mu_1 \cdots \mu_p} + (1/2)(C\Gamma^{\mu_1 \cdots \mu_p})_{\alpha\beta} \theta^\alpha \Pi^\beta \tag{5}$$

is LI. These new parameters $\varphi^{\mu_1 \cdots \mu_p}$ generalize the spacetime parameters x^μ, and their associated generators $Z_{\mu_1 \ldots \mu_p}$ may be considered as *generalised momenta*. There are no (two-cocycle) restrictions coming from the Jacobi identity since the r.h.s of (4) is trivially consistent with $d(d\Pi^{\mu_1 \cdots \mu_p}) \equiv 0$.

2) *Group extension point of view*

The closedness of the r.h.s. of (4) means that the Lorentz tensor-valued two-cocycle on sTr_D, $\xi^{\mu_1 \cdots \mu_p}(\theta', \theta) = \theta'^\alpha (C\Gamma^{\mu_1 \cdots \mu_p})_{\alpha\beta} \theta^\beta$, satisfies also (trivially) the *two-cocycle condition*

$$\xi(\theta, \theta')^{\mu_1 \cdots \mu_p} + \xi(\theta + \theta', \theta'')^{\mu_1 \cdots \mu_p} = \xi(\theta, \theta' + \theta'')^{\mu_1 \cdots \mu_p} + \xi(\theta', \theta'')^{\mu_1 \cdots \mu_p} \tag{6}$$

The symmetry of $(C\Gamma^{\mu_1 \cdots \mu_p})_{\alpha\beta}$ is needed to prevent the above two-cocycle from being trivial, since the possible function $\eta^{\mu_1 \cdots \mu_p}(\theta)$ on sTr_D that might generate the two-coboundary $(\xi_{cob}^{\mu_1 \cdots \mu_p}(\theta', \theta) = \eta^{\mu_1 \cdots \mu_p}(\theta + \theta') - \eta^{\mu_1 \cdots \mu_p}(\theta') - \eta^{\mu_1 \cdots \mu_p}(\theta))$ is zero: $\eta^{\mu_1 \cdots \mu_p}(\theta) = \theta^\alpha (C\Gamma^{\mu_1 \cdots \mu_p})_{\alpha\beta} \theta^\beta \equiv 0$. Hence, *The problem of finding all* central *extensions of the sTr_D algebra $\{D_\alpha, D_\beta\} = 0$ is reduced to finding a basis of the symmetric space $\Pi^{(\alpha} \otimes \Pi^{\beta)}$ in terms of p-Lorentz tensors $(C\Gamma^{\mu_1 \cdots \mu_p})_{\alpha\beta}$ symmetric in (α, β).*

The answer for different spacetime dimensions D depends on the properties of their respective Γ matrices, since they determine the existence of non-trivial cocycles (see [13] for a table). We shall only consider here the example of the

2.1 D=11, M-theory extended superspace

The maximally centrally extended FDA is obtained by adding Π^μ, $\Pi^{\mu_1 \mu_2}$, $\Pi^{\mu_1 \cdots \mu_5}$ to $\Pi^\alpha = d\theta^\alpha$ (θ^α Majorana, $\alpha = 1, \ldots, 32$) satisfying

$$\begin{aligned} &d\Pi^\alpha = 0 \; , \; d\Pi^\mu = \frac{1}{2}(C\Gamma^\mu)_{\alpha\beta} \Pi^\alpha \Pi^\beta \quad , \\ &d\Pi^{\mu_1 \mu_2} = \frac{1}{2}(C\Gamma^{\mu_1 \mu_2})_{\alpha\beta} \Pi^\alpha \Pi^\beta \; , \; d\Pi^{\mu_1 \ldots \mu_5} = \frac{1}{2}(C\Gamma^{\mu_1 \ldots \mu_5})_{\alpha\beta} \Pi^\alpha \Pi^\beta \quad . \end{aligned} \tag{7}$$

There are no *one*-forms $\Pi^{\mu_1 \mu_2}$, $\Pi^{\mu_1 \cdots \mu_5}$ LI on Σ. They can be made LI by introducing *new* 'central' (tensorial) coordinates $\varphi^{\mu_1 \mu_2}$, $\varphi^{\mu_1 \cdots \mu_5}$. These *define* the $D = 11$, $N = 1$ extended superspace

$\tilde{\Sigma}(\theta^\alpha, x^\mu, \varphi^{\mu_1\mu_2}, \varphi^{\mu_1\ldots\mu_5})$. In terms of the central generators $X_\mu = \partial/\partial x^\mu$, $Z_{\mu_1\mu_2} = \partial/\partial\varphi^{\mu_1\mu_2}$, $Z_{\mu_1\ldots\mu_5} = \partial/\partial\varphi^{\mu_1\ldots\mu_5}$, the $D=11$ *supersymmetry M-algebra* dual to (7) is

$$\{D_\alpha, D_\beta\} = (C\Gamma^\mu)_{\alpha\beta}X_\mu + (C\Gamma^{\mu_1\mu_2})_{\alpha\beta}Z_{\mu_1\mu_2} + (C\Gamma^{\mu_1\ldots\mu_5})_{\alpha\beta}Z_{\mu_1\ldots\mu_5} \quad . \tag{8}$$

This is usually referred to [15] as the *M-theory superalgebra*. The group law of the extended superspace $\tilde{\Sigma}(\theta^\alpha, x^\mu, \varphi^{\mu_1\mu_2}, \varphi^{\mu_1\ldots\mu_5})$ is obtained easily as the simplest Σ case (*cf.* (2)).

For a recent discussion of the M-algebra, in which the tensorial central charges are considered as bilinears of spinors, see [16].

3 Non-central additional generators and their extended superspaces

The above are *central* extensions of the basic odd abelian algebra $\{D_\alpha, D_\beta\} = 0$ by *bosonic* tensorial generators. But there are also extensions by fermionic generators that make non-abelian *e.g.*, the $[X_\mu, D_\alpha] = 0$ commutator. The CE cohomology analysis is also useful here. Let us start from a centrally extended superspace $\tilde{\Sigma}(\theta^\alpha, x^\mu, \varphi_{\mu_1\ldots\mu_p})$, p fixed, and LI one-forms Π^μ, Π^α, $\Pi_{\mu_1\ldots\mu_p}$, satisfying the MC eqs.

$$d\Pi^\mu = a_s(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta \quad , \quad d\Pi_{\mu_1\ldots\mu_p} \equiv a_0(C\Gamma_{\mu_1\ldots\mu_p})_{\alpha\beta}\Pi^\alpha\Pi^\beta \quad , \tag{9}$$

where a_s, a_0 are not fixed for convenience. A non-trivial CE two-cocycle with p indices has to be of the type $(\mu_1 \ldots \mu_{p-1}\alpha_1)$ and, hence, the only available LI *two*-forms (in this case, fermionic) are

$$\rho^{(1)}_{\mu_1\ldots\mu_{p-1}\alpha_1} = (C\Gamma_{\nu\mu_1\ldots\mu_{p-1}})_{\beta\alpha_1}\Pi^\nu\Pi^\beta \ , \ \rho^{(2)}_{\mu_1\ldots\mu_{p-1}\alpha_1} = (C\Gamma^\nu)_{\beta\alpha_1}\Pi_{\nu\mu_1\ldots\mu_{p-1}}\Pi^\beta \quad . \tag{10}$$

For $p=1$, both are closed. For $p \geq 2$, the condition $d(\rho^{(1)} + \lambda_2\rho^{(2)}) = 0$ fixes $\lambda_2 = a_s/a_0$ provided

$$(C\Gamma^\nu)_{(\alpha\beta}(C\Gamma_{\nu\mu_1\ldots\mu_{p-1}})_{\gamma\delta)} = 0 \quad , \tag{11}$$

which holds for the (D,p) of the scalar branescan [7]. Condition (11) is a *new* feature of the *'non-central' case*; in the central (bosonic two-cocycles) case, the closedness was *trivially* satisfied and hence there was *no condition on D; only* p (the rank p of the Lorentz tensor, see below (5)) was restricted by the $(\alpha\beta)$ symmetry of the tensor. We may introduce now a new one–form $\Pi_{\mu_1\ldots\mu_{p-1}\alpha_1}$ with

$$d\Pi_{\mu_1\ldots\mu_{p-1}\alpha_1} = a_1\left((C\Gamma_{\nu\mu_1\ldots\mu_{p-1}})_{\beta\alpha_1}\Pi^\nu\Pi^\beta + \frac{a_s}{a_0}\,(C\Gamma^\nu)_{\beta\alpha_1}\Pi_{\nu\mu_1\ldots\mu_{p-1}}\Pi^\beta)\right) \tag{12}$$

(for $p=1$ the coefficient of the second term can be arbitrary). This MC equation implies that both $[D_\alpha, X_\mu]$ and $[D_\alpha, Z^{\mu_1\ldots\mu_p}]$ are modified by a term proportional to $Z^{\mu_1\ldots\mu_{p-1}\alpha_1}$, the latter being the only central generator at this stage ($Z^{\mu_1\ldots\mu_{p-1}\alpha_1}$ is central because, by construction, $\Pi_{\mu_1\ldots\mu_{p-1}\alpha_1}$ cannot appear at the r.h.s. of a MC equation expressing the differential of a LI form).

The general features of the extensions with non-central fermionic generators are:

a) The extension two-cocycles (two-forms) may be *fermionic* (eqs. (10)). This leads to non-zero [bosonic,fermionic] commutators.

b) At any stage in the chain of extensions, the only *central* generator present is the one introduced in the *last* extension.

c) Successive central extensions substitute one spinorial index for a vectorial one. This leads to one-forms of the type

$$\Pi_{\mu_1\ldots\mu_{p-k}\alpha_1\ldots\alpha_k} \equiv \Pi_{\rho_k} \ , \quad \rho_k \equiv (\mu_1 \ldots \mu_{p-k}\alpha_1 \ldots \alpha_k) \quad , \tag{13}$$

where ρ_k labels the additional coordinates of the extended superspace $\tilde{\Sigma}$.

d) The procedure ends when the p vector indices have become spinorial ones so that $\Pi_{\rho_k} \to \Pi_{\rho_p} \equiv \Pi_{\alpha_1\ldots\alpha_p}$.

e) For a given p, there are consistency conditions that restrict the spacetime dimension D; for instance, the Green algebra exists for D=3,4,6 and 10 only [5].

All the extended superspaces have a natural fibre bundle structure that is inherited from their group extension character; we refer to [13] (see also [14]) for details.

3.1 Two applications: the GS superstring and the supermembrane

Consider the Green-Schwarz superstring case (p=1, D=10, N=1). We shall denote by φ_μ the additional vector parameter and by Z^μ, $\Pi^{(\varphi)}_\mu$ the associated generator and LI form [13]. The MC eqs. are

$$\begin{aligned} d\Pi^\alpha &= 0\ , \quad d\Pi^\mu = (1/2)(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta\ , \\ d\Pi^{(\varphi)}_\mu &= (1/2)(C\Gamma_\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta\ ,\ d\Pi_\alpha = (C\Gamma_\mu)_{\alpha\beta}\Pi^\mu\Pi^\beta + (C\Gamma^\mu)_{\alpha\beta}\Pi^{(\varphi)}_\mu\Pi^\beta\ ; \end{aligned} \tag{14}$$

$\mu = 0,\dots,9$, and all spinors here are MW ($\theta^\alpha \equiv \mathcal{P}_+\theta^\alpha$, $\Pi^\alpha \equiv \mathcal{P}_+ d\theta^\alpha$; notice that Π^α and Π_α are unrelated). The two terms in the r.h.s. of the last of (14) are individually closed ($d(d\Pi_\alpha) = 0$ follows from (11) for $p = 1$, *i.e.* by $(C\Gamma^\mu)_{(\alpha\beta}(C\Gamma_\mu)_{\gamma\delta)} = 0$) and hence their relative normalization cannot be fixed by requiring $d(d\Pi_\alpha) = 0$.

The corresponding Lie superalgebra contains an additional *fermionic* central generator, Z^β, and is given by

$$\begin{aligned} \{D_\alpha, D_\beta\} &= (C\Gamma^\mu)_{\alpha\beta}X_\mu + (C\Gamma_\mu)_{\alpha\beta}Z^\mu\ , \\ [D_\alpha, X_\mu] &= (C\Gamma_\mu)_{\alpha\beta}Z^\beta\ , \quad [D_\alpha, Z^\mu] = (C\Gamma^\mu)_{\alpha\beta}Z^\beta\quad ; \end{aligned} \tag{15}$$

if one omits Z^μ, it reduces to the *Green algebra* [5][2]. Note that X_μ *is no longer central* due to the presence of Z^β. The associated group manifold is the *GS superstring extended superspace* $\tilde\Sigma(\theta^\alpha, x^\mu, \varphi_\mu, \varphi_\alpha)$; its group law is given and discussed in [13].

As mentioned, the extended superspaces are suitable to define manifestly invariant WZ terms. For instance, using the LI forms on $\tilde\Sigma(\theta^\alpha, x^\mu, \varphi_\mu, \varphi_\alpha)$, one obtains the *manifestly invariant WZ term for the GS superstring* (*cf.* [8])

$$S_{WZ} = \int_W \phi^*(\tilde b) = \int_W \phi^*(\Pi^{(\varphi)}_\mu\Pi^\mu + \frac{1}{2}\Pi_\alpha\Pi^\alpha)\quad , \tag{16}$$

$d\tilde b = db = h = (C\Gamma_\mu)_{\alpha\beta}\Pi^\mu\Pi^\alpha\Pi^\beta$ and hence $\phi^*(\tilde b)$ and the standard WZ term $\phi^*(b)$ are equivalent; b and $\tilde b$ *differ only by an exact form.*

Similarly, it is also possible to write a manifestly invariant D=11 membrane (p=2) WZ term. It exists on the $D = 11$ *supermembrane extended superspace group* $\tilde\Sigma(\theta^\alpha, x^\mu, \varphi_{\mu\nu}, \varphi_{\mu\alpha}, \varphi_{\alpha\beta})$, and is found to be

$$\tilde b = (2/3)\Pi_{\mu\nu}\Pi^\mu\Pi^\nu - (3/5)\Pi_{\mu\alpha}\Pi^\mu\Pi^\alpha - (2/15)\Pi_{\alpha\beta}\Pi^\alpha\Pi^\beta\ , \tag{17}$$

as given in [6]. Again, $\tilde b$ *depends on the additional variables* φ *through total differentials* since $d\tilde b = db = h = (C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\mu\Pi^\nu\Pi^\alpha\Pi^\beta$. The non-WZ part of the action does not depend on the additional variables of extended superspace, and remains the standard one. As we shall see, this situation will change for D-branes.

4 New Noether currents and charges

Let us now give the general expression for the Noether currents for the additional symmetries $j^i_{\sigma_l}$ (see (13) for the notation) using the manifestly invariant WZ forms $\tilde{\mathcal{L}}_{WZ}$ defined on the various extended superspaces $\tilde\Sigma$. The Lagrangian $\tilde{\mathcal{L}}_{WZ}(\xi)$ on the worldvolume W (of coordinates ξ^i, $i = 0, 1, \dots, p$) is the pull-back $\phi^*(\tilde{\mathcal{L}}_{WZ}) = \tilde{\mathcal{L}}_{WZ}(\xi)d^{p+1}\xi$ of the (p+1)-form $\tilde{\mathcal{L}}_{WZ}$ on $\tilde\Sigma$ by the map $\phi : W \longrightarrow \tilde\Sigma$. The charges that correspond to the current densities $j^i_{\sigma_l}(\xi)$ appear on the r.h.s. of the supersymmetry algebra.

The WZ part of the action is $\int \tilde{\mathcal{L}}_{WZ}(\xi)d^{p+1}\xi$ and, *since only* $\tilde{\mathcal{L}}_{WZ}$ *depends on the additional variables of the extended superspace* $\tilde\Sigma$ (different from (x^μ, θ^α) of Σ), we shall focus on $\tilde{\mathcal{L}}_{WZ}$. We find first the general expression for the Noether currents and then apply it to the simple cases of the GS superstring and the supermembrane.

[2]This algebra may be viewed as a 'stabilising deformation' of Σ [17]. In this context, stability is achieved by exhausting the second Lie algebra cohomology group (the non-trivial two-cocycle space) *i.e.*, by extending maximally under certain conditions, as done in [13]. This means, *e.g.*, including *both* generators $Z_{\mu\nu}$ and $Z_{\mu_1\dots\mu_5}$ in (8) if only bosonic ones are considered (*cf.* [9]), and similarly for the other cases.

We start by writing the manifestly invariant density $\tilde{\mathcal{L}}_{WZ}(\xi)$ as

$$\tilde{\mathcal{L}}_{WZ}(\xi) \equiv \Pi_{\rho_k i}(\xi)\,\Lambda^{\rho_k i}(\xi) \quad , \quad \xi^i=(\tau,\sigma) \quad , \quad i=0,1,\ldots,p \quad , \tag{18}$$

(see (13)) where Λ^{ρ_k} is defined by (18) and denotes the LI (p-)form

$$\Lambda^{\rho_k} \equiv \Lambda^{\mu_1\ldots\mu_{p-k}\alpha_1\ldots\alpha_k} = a_k \Pi^{\mu_1}\ldots\Pi^{\mu_{p-k}}\Pi^{\alpha_1}\ldots\Pi^{\alpha_k} \quad , \tag{19}$$

$\Pi_{\rho_k i}$=$(\phi^*(\Pi_{\rho_k})_i$ and $\Lambda^{\rho_k i}$ corresponds to $a_k\epsilon^{ij_1\ldots j_p}\Pi^{\mu_1}_{j_1}\ldots\Pi^{\mu_{p-k}}_{j_{p-k}}\Pi^{\alpha_1}_{j_{p-k+1}}\ldots\Pi^{\alpha_k}_{j_p}$ (the constants a_k are fixed by $d\tilde{b}=h$).

Given the group law $g''=g'g$ ($g''^A=g''^A(g',g)$, $A=(\alpha,\mu;\rho_k)$) of $\tilde{\Sigma}$, the LI one-forms $\Pi^A(g)$ and the RI vector fields $Z_A(g)$ are given by

$$\Pi^A(g)=\Pi^A_B(g)\,dg^B = \left.\frac{\partial g''^A(g',g)}{\partial g^B}\right|_{g'=g^{-1}} dg^B \;, \quad Z_A(g) = \left.\frac{\partial g''^D(g',g)}{\partial g'^A}\right|_{g'=e}\frac{\partial}{\partial g^D} \tag{20}$$

(see, *e.g.*, [10]). The Z_A generate the left g^A-translations of $\tilde{\Sigma}$.

The $\tilde{\mathcal{L}}_{WZ}(\xi)$ *contribution to* $j^i_A(\xi)=j^i_{A(kin)}(\xi)+j^i_{A(WZ)}(\xi)$ is

$$j^i_{A(WZ)}(\xi)=(\delta_A g^B)\frac{\partial\tilde{\mathcal{L}}_{WZ}}{\partial g^B{}_{,i}} \equiv (Z_A.g^B)\frac{\partial\tilde{\mathcal{L}}_{WZ}}{\partial g^B{}_{,i}} \quad . \tag{21}$$

Let the extended superspace index refer to a new coordinate, $A=\sigma_l$, and let us compute $j^i_{\sigma_l}$. Since only $\tilde{\mathcal{L}}_{WZ}$ depends on the new coordinates, $j^i_{\sigma_l(kin)}=0$. The B summation in (21) is reduced to a summation over the additional coordinates index η_k since the vector fields Z_{σ_l} do not have $\partial/\partial x^\mu$, $\partial/\partial\theta^\alpha$ components and thus $Z_{\sigma_l}.g^B=0$ for $g^B=(\theta^\alpha,x^\mu)$. Moreover, since $\Lambda^{\rho_k}=\Lambda^{\rho_k}(\Pi^\mu,\Pi^\alpha)$ and Π^μ,Π^α are defined on the standard Σ, the $\Lambda^{\rho_k i}$ part does not depend on φ_{η_k} (g^{η_k} in (21)),

$$j^i_{\sigma_l}=(Z_{\sigma_l}.g^{\eta_k})\left(\frac{\partial}{\partial g^{\eta_k}{}_{,i}}\Pi_{\rho_k i}\right)\Lambda^{\rho_k i} \quad . \tag{22}$$

Using Eq. (20),

$$j^i_{\sigma_l}=(Z_{\sigma_l}.g^{\eta_k})\left\{\frac{\partial}{\partial g^{\eta_k}{}_{,i}}\left(\left.\frac{\partial g''_{\rho_k}(g',g)}{\partial g^B}\right|_{g'=g^{-1}} g^B{}_{,i}\right)\right\}\Lambda^{\rho_k i} = (Z_{\sigma_l}.g^{\eta_k})\left.\frac{\partial g''_{\rho_k}(g',g)}{\partial g^{\eta_k}}\right|_{g'=g^{-1}}\Lambda^{\rho_k i} \tag{23}$$

since $g''\neq g''(g_{,i})$. This gives the *general expression for the Noether currents associated with the additional generators*:

$$j^i_{\sigma_l}=(Z_{\sigma_l}.g''_{\rho_k}(g',g)|_{g'=g^{-1}})\Lambda^{\rho_k i}\equiv T_{\sigma_l\rho_k}\Lambda^{\rho_k i} \quad , \tag{24}$$

where T corresponds to the adjoint representation $Ad(g^{-1})$ and depends on ξ through $g(\xi)$ (notice that if X^R is RI and Π^L as a LI one-form, $i_{X^R}\Pi^L=Ad(g^{-1})X^R$). Since for $A=\sigma_l$ we may restrict D to η_k ($Z^{\mu,\alpha}_{\sigma_l}(g)=0$), eq. (24) may also be written as

$$j^i_{\sigma_l}(\xi)=\left(\left.\frac{\partial g''^{\eta_k}(g',g)}{\partial g'^{\sigma_l}}\right|_{g'=e}\left.\frac{\partial g''_{\rho_k}(g',g)}{\partial g^{\eta_k}}\right|_{g'=g^{-1}}\right)\Lambda^{\rho_k i} \tag{25}$$

using (20); the bracketed term is determined by the group $\tilde{\Sigma}$ only, and $\Lambda^{\rho_k i}$ by $\tilde{\mathcal{L}}_{WZ}(\xi)$.

a) *D=10, N=1 superstring*:

Using expression (16) for (18), we find that the conserved Noether currents are

$$j^{\mu i}_{(\varphi)}=\epsilon^{ij}\partial_j x^\mu \quad , \quad j^{\alpha i}=(1/2)\epsilon^{ij}\partial_j\theta^\alpha \;, \tag{26}$$

and the charges [18]

$$Z^\mu=\oint d\sigma j^{\mu 0}_{(\varphi)}=\oint d\sigma\,\frac{\partial x^\mu}{\partial\sigma} \;, \quad Z^\alpha=\oint d\sigma j^{\alpha 0}=\oint d\sigma\frac{1}{2}\frac{\partial\theta^\alpha}{\partial\sigma}=0 \;, \tag{27}$$

assuming that θ is periodic in σ (*cf.* [19]). It is clear that, in general, the integral of j^0 (as, *e.g.*, for $j^{\mu 0}_{(\varphi)}$) leads to a non-zero result if the topology is nontrivial (the loop is not contractible).

b) *D=11 2-brane*:

It can be shown from eq. (17) that the currents can be written as the worldvolume duals of the *current two-forms*

$$
\begin{aligned}
J^{\mu\nu} &= d\left(\frac{2}{3}x^{[\mu}dx^{\nu]} + \frac{1}{15}\theta^\alpha x^{[\mu}(C\Gamma^{\nu]})_{\alpha\beta}d\theta^\beta\right) \quad , \\
J^{\kappa\alpha} &= d\left(\frac{3}{5}dx^\kappa\theta^\alpha - \frac{1}{30}(C\Gamma^\kappa)_{\beta\gamma}\theta^\beta\theta^\alpha d\theta^\gamma\right) , \quad J^{\beta\gamma} = d(-\frac{2}{15}\theta^\beta d\theta^\gamma) \quad ;
\end{aligned}
\tag{28}
$$

current conservation follows from $dJ = 0$. For periodic θ's the charges $Z^{\kappa_1\alpha_1}$, $Z^{\beta_1\gamma_1}$ turn out to be zero, but not $Z^{\mu_1\nu_1}$ for a non-trivial closed two-cycle [18] (in the general p-case, the integrals are over non-trivial de Rham p-cycles; we refer to [18] for details on topological charges). Thus, the above assumptions provide a realization of the extended algebra where only the bosonic $Z^{\mu\nu}$ generator is realized non-trivially.

5 The case of D-branes

Consider first a bosonic background such that the action of the Dp-brane [20, 21] reduces to

$$
I = \int d^{p+1}\xi\sqrt{-\det(\partial_i x^\mu \partial_j x_\mu + F_{ij})} \quad , \tag{29}
$$

where $F = dA$ and $A(\xi) = A_i(\xi)d\xi^i$ is the worldvolume Born-Infeld (BI) field.

Let us look for a *manifestly supersymmetric generalisation*. This means substituting first Π^μ_i for $\partial_i x^\mu$, $F_{ij} = \partial_{[i}A_{j]}$ by $\mathcal{F} = dA - B$, and then adding a WZ term b, $db = h$. A previous analysis [12] of the WZ terms of the scalar branescan [7] showed that *WZ terms may be characterized and classified by CE-(p+2)cocycles.* The same philosophy is successful for the Dp-branes. The result is that Dp-branes may also be be characterized (see below and [13] for details and further references) by means of non-trivial CE $(p+2)$-cocycles, recovering Polchinski's consistency conditions [20] (p even/odd for IIA/IIB). In the case of D-branes, however, and due to the presence of F_{ij} in the kinetic term (29) the situation turns out to be different from that of the previous p-branes: the new variables will appear in the action *non-trivially*, not as total derivatives.

5.1 Example: the D2-brane defined on its extended superspace

Consider the D2-brane. The starting point is now the IIA-type FDA plus the $d\mathcal{F}$ equation *i.e.*

$$
\begin{aligned}
d\Pi^\alpha &= 0 & d\Pi^\mu &= \tfrac{1}{2}(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta \\
d\Pi &= \tfrac{1}{2}(C\Gamma_{11})_{\alpha\beta}\Pi^\alpha\Pi^\beta & d\Pi_{\mu\nu} &= \tfrac{1}{2}(C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\alpha\Pi^\beta \\
d\Pi^{(z)}_\mu &= \tfrac{1}{2}(C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\alpha\Pi^\beta & d\mathcal{F} &= (C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\mu\Pi^\alpha\Pi^\beta \, ,
\end{aligned}
\tag{30}
$$

($\mu = 0, \ldots 9$, $\alpha = 1, \ldots 32$). This is justified *e.g* by the fact that the dual of the first 5 eqs. is the algebra obtained when one computes the algebra of Noether charges for the type IIA D2-brane [22]. The next step is extending this algebra with the generators obtained by replacing vector indices by spinorial ones, as outlined after (12). In the case of the D2-brane this is not difficult to do because, apart from the equation for $d\mathcal{F}$, the FDA above is actually the dimensional reduction to D=10 of the D=11 one (eq. (7) with generators with one or two vector indices since p=2),

$$
d\Pi^{\tilde\mu} = (1/2)(C\Gamma^{\tilde\mu})_{\alpha\beta}\Pi^\alpha\Pi^\beta \, , \qquad d\Pi_{\tilde\mu\tilde\nu} = (1/2)(C\Gamma_{\tilde\mu\tilde\nu})_{\alpha\beta}\Pi^\alpha\Pi^\beta \, , \tag{31}
$$

where ($\tilde\mu = (\mu, 10) = 0, 1, \ldots 10$), and in which one sets $\Pi^{\tilde\mu} \equiv (\Pi^\mu, \Pi^{10} \equiv \Pi)$, $\Pi_{\tilde\mu\tilde\nu} \equiv (\Pi_{\mu\nu}, \Pi_{\mu 10} \equiv \Pi^{(z)}_\mu)$. This $D = 11$ FDA may be extended. The $D = 10$ dimensional reduction of the extended algebra gives

$$
\begin{aligned}
d\Pi^\alpha &= 0\,, \quad d\Pi^\mu = \frac{1}{2}(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta\,, \quad d\Pi = \frac{1}{2}(C\Gamma_{11})_{\alpha\beta}\Pi^\alpha\Pi^\beta\,,\\
d\Pi_{\mu\nu} &= \frac{1}{2}(C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\alpha\Pi^\beta\,, \quad d\Pi^{(z)}_\mu = \frac{1}{2}(C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\alpha\Pi^\beta\,,\\
d\Pi_{\mu\alpha} &= (C\Gamma_{\nu\mu})_{\alpha\beta}\Pi^\nu\Pi^\beta + (C\Gamma_{11}\Gamma_\mu)_{\alpha\beta}\Pi\Pi^\beta + (C\Gamma^\nu)_{\alpha\beta}\Pi_{\nu\mu}\Pi^\beta - (C\Gamma_{11})_{\alpha\beta}\Pi^{(z)}_\mu\Pi^\beta \quad ,\\
d\Pi^{(z)}_\alpha &= (C\Gamma_\nu\Gamma_{11})_{\alpha\beta}\Pi^\nu\Pi^\beta + (C\Gamma^\nu)_{\alpha\beta}\Pi^{(z)}_\nu\Pi^\beta \quad ,\\
d\Pi_{\alpha\beta} &= -\frac{1}{2}(C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\mu\Pi^\nu - (C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\mu\Pi - \frac{1}{2}(C\Gamma^\mu)_{\alpha\beta}\Pi_{\mu\nu}\Pi^\nu\\
&\quad +\frac{1}{2}(C\Gamma_{11})_{\alpha\beta}\Pi^{(z)}_\mu\Pi^\mu - \frac{1}{2}(C\Gamma^\mu)_{\alpha\beta}\Pi^{(z)}_\mu\Pi + \frac{1}{4}(C\Gamma^\mu)_{\alpha\beta}\Pi_{\mu\delta}\Pi^\delta\\
&\quad +\frac{1}{4}(C\Gamma_{11})_{\alpha\beta}\Pi^{(z)}_\delta\Pi^\delta + 2\Pi_{\mu(\beta}(C\Gamma^\mu)_{\alpha)\delta}\Pi^\delta + 2(C\Gamma_{11})_{\delta(\alpha}\Pi^{(z)}_{\beta)}\Pi^\delta \quad .
\end{aligned}
\tag{32}
$$

Using the new forms it is possible to find a manifestly invariant WZ form $\tilde{b}$, $d\tilde{b} = h$; h is given by

$$
h = (C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\mu\Pi^\nu\Pi^\alpha\Pi^\beta - (C\Gamma_{11})_{\alpha\beta}\Pi^\alpha\Pi^\beta\mathcal{F} \quad , \tag{33}
$$

and the *manifestly invariant WZ term for the type IIA D2-brane* by

$$
\tilde{b} = \frac{2}{3}\Pi_{\mu\nu}\Pi^\mu\Pi^\nu + \frac{4}{3}\Pi^{(z)}_\mu\Pi^\mu\Pi - \frac{2}{15}\Pi_{\alpha\beta}\Pi^\alpha\Pi^\beta - \frac{3}{5}\Pi_{\mu\alpha}\Pi^\mu\Pi^\alpha - \frac{3}{5}\Pi^{(z)}_\alpha\Pi\Pi^\alpha - 2\Pi\mathcal{F}\,. \tag{34}
$$

We expect that this analysis also holds true for the other values of p.

The extended free differential algebra is not the dual of a Lie algebra because it includes the equation for the *three*-form $d\mathcal{F}$. However,

$$
d(\frac{1}{2}\Pi^\alpha\Pi^{(z)}_\alpha - \Pi^\mu\Pi^{(z)}_\mu) = (C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\Pi^\mu\Pi^\alpha\Pi^\beta \tag{35}
$$

so that, on the extended superspace $\tilde{\Sigma}(\theta^\alpha, x^\mu, \varphi_\mu, \varphi_\alpha)$ we may set

$$
\mathcal{F} = (1/2)\Pi^\alpha\Pi^{(z)}_\alpha - \Pi^\mu\Pi^{(z)}_\mu \quad . \tag{36}
$$

Since $\mathcal{F} = dA - B$ *and* B *is defined on* Σ, it follows that dA may be written on $\tilde{\Sigma}$. Making use of the explicit form of the LI one-forms in terms of the extended superspace variables, it is easy to identify A *as the one-form on* $\tilde{\Sigma}$

$$
A = \varphi_\mu dx^\mu + (1/2)\varphi_\alpha d\theta^\alpha \quad . \tag{37}
$$

In the present approach, the customary BI worldvolume field $A_i(\xi)d\xi^i$ becomes $\phi^*(A)$; we might even say that the existence of the BI field is a consequence of supersymmetry. We now check the consistency of the replacement (37).

a) The Euler Lagrange equations are still the same. Let $I[x^\mu(\xi), \theta^\alpha(\xi), A_i(\xi)]$ be the action before making the substitution. The EL equations are

$$
\delta I/\delta x^\mu = 0\,, \qquad \delta I/\delta\theta^\alpha = 0\,, \qquad \delta I/\delta A_j = 0\,. \tag{38}
$$

When the substitution is made,

$$
\begin{aligned}
\int d\xi'^{p+1}\frac{\delta I}{\delta A_j(\xi')}\frac{\delta A_j(\xi')}{\delta x^\mu(\xi)} + \frac{\delta I}{\delta x^\mu} &= 0, \qquad \frac{\delta I}{\delta\varphi_\mu} = \frac{\delta I}{\delta A_j}\partial_j x^\mu = 0\\
\int d\xi'^{p+1}\frac{\delta I}{\delta A_j(\xi')}\frac{\delta A_j(\xi')}{\delta\theta^\alpha(\xi)} + \frac{\delta I}{\delta\theta^\alpha} &= 0, \qquad \frac{\delta I}{\delta\varphi_\alpha} = \frac{1}{2}\frac{\delta I}{\delta A_j}\partial_j\theta^\alpha = 0\,.
\end{aligned}
\tag{39}
$$

We see that to avoid the collapse of one or more worldvolume dimensions we must have $\delta I/\delta A_j = 0$ which implies eqs. (38). This also follows from the fact that $\delta I/\delta\varphi_\mu = 0$ implies $(\delta I/\delta A_j)g_{ij} = 0$, where $g_{ij} \equiv \Pi^\mu_i\Pi_{\mu j} = \partial_i x^\mu\partial_j x_\mu +$(nilpotent terms) is the induced worldvolume metric. Thus, we must have $\delta I/\delta A_j = 0$ to prevent g_{ij} from being degenerate. As a result, $\delta I/\delta\varphi_\alpha = 0$ is satisfied identically and it is a Noether identity.

b) The gauge transformations of $A_i(\xi)$ can be reinterpreted in the new language. If one defines $\delta\varphi_\mu = \partial_\mu\lambda$ and $\delta\varphi_\alpha = 2\partial_\alpha\lambda$, by means of a superfield λ such that $\phi^*\lambda(x^\mu,\theta^\alpha) = \Lambda(\xi)$, then $\phi^*(A)$ behaves as expected: $\delta(\phi^*[\varphi_\mu dx^\mu + \frac{1}{2}\varphi_\alpha d\theta^\alpha]) = \partial_i\Lambda$.

c) The number of worldvolume degrees of freedom remains the same. Let us first note that, since $\delta I/\delta\varphi_\alpha = 0$ is a Noether identity, the second Noether theorem tells us that there exists a gauge symmetry that can be used to set $\varphi_\alpha = 0$. Thus, the 'physical' part of A is contained in $\varphi_\mu dx^\mu$. The identification (37) is therefore equivalent to replacing $A_i(\xi)$ by $\varphi_\mu(\xi)\partial_i x^\mu(\xi)$. We now notice that the D equations $\delta I/\delta\varphi_\mu = 0$ produce only $(p+1)$ independent ones, $\delta I/\delta A_j = 0$; the remaining $D-(p+1)$ equations are Noether identities that reflect the existence of further gauge symmetries. To check explicitly the degrees of freedom we first adopt the gauge $(x^0(\xi) = \tau,\ x^1(\xi) = \xi^1, ..., x^p(\xi) = \xi^p)$. Then,

$$(\phi^* A)_i = \varphi_\mu(\xi)\partial_i x^\mu(\xi) = \varphi_i(\xi) + \varphi_K(\xi)\partial_i x^K(\xi)\ , \quad K = p+1, \ldots, D-1\ . \tag{40}$$

We see that, apparently, we are describing the $(p+1)$ components A_i of the BI field using D functions (φ_i, φ_K). The mismatch in the number of degrees of freedom is sorted out by the existence of the bosonic gauge symmetries that allow us to remove the additional $(D-p-1)$ functions. Futhermore, since the components φ_μ enter the action non-trivially only through $(\phi^* A)_i$, any local transformation of $\varphi_\mu(\xi)$ that leaves $(\phi^* A)_i$ unchanged will be a gauge symmetry of the action. Consider then

$$\delta\varphi_i(\xi) = -\alpha_K(\xi)\partial_i x^K(\xi)\ , \quad \delta\varphi_K(\xi) = \alpha_k(\xi)\ . \tag{41}$$

This specific transformation has the property $\delta(\phi^* A)_i = 0$, so it is a gauge symmetry that can be used to set $\varphi_K = 0$ by taking $\alpha_k = \varphi_K$, so $\phi^*(A)_i = \varphi_i$. Hence, we may identify $A_i = \phi^*(A)_i$.

For the IIB Dp-brane, an analysis similar to that in this section (for odd p) can be made. In fact, the origin of $A(\xi)$ in the p=1 IIB D-string case was discussed in [23] (see also [24]) by introducing an appropriate extended group manifold. We may conclude, then, that *the different worldvolume fields are introduced naturally through the pull-back of coordinates (forms) of (defined on) suitably extended superspaces.*

6 Noether charges and D-brane actions

The worldvolume field $A(\xi)$ that appears in the D2-brane action may be written in terms of the variables of the superstring extended superspace $\tilde{\Sigma}(x^\mu, \theta^\alpha, \varphi_\mu, \varphi_\alpha)$. The D2-WZ term, which is quasi-invariant in these coordinates, can be made strictly invariant by further extending the previous superspace to $\tilde{\Sigma} = (x^\mu, \theta^\alpha, \varphi_\mu, \varphi_\alpha,\ \varphi_{\mu\nu}, \varphi_{\mu\alpha}, \varphi_{\alpha\beta}, \varphi)$. In this way, the whole action is invariant. The canonical commutators of the charges generating the symmetries of the action (denoted by a hat) give a realization of the 'right' version of the 'left' Lie algebra dual to (32).

Consider the $\{Q_\alpha, Q_\beta\}$ commutator, that we shall write as

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^\mu)_{\alpha\beta}P_\mu + (C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\hat{Z}^\mu + (C\Gamma_{\mu\nu})_{\alpha\beta}\hat{Z}^{\mu\nu} + (C\Gamma_{11})_{\alpha\beta}\hat{Z} \quad . \tag{42}$$

With $A = A(\xi)$, the $C\Gamma_{\mu\nu}$ and $C\Gamma_{11}$ contributions would come from the quasi-invariance of the WZ Lagrangian, while $C\Gamma_{11}\Gamma_\mu$ would be the result of the contribution of the $A(\xi)$ field to the Noether current [22] (see also [25]). This is because the supersymmetry transformations *do not close on* A, and this produces an additional term by a mechanism similar to the one in the standard quasi-invariance case.

These modifications become transparent by formulating the action on the extended superspace [13]. Consider the formulation of the D2-brane on the extended superspace with quasi-invariant WZ term $b = b(x^\mu, \theta^\alpha, \varphi_\mu, \varphi_\alpha)$. The conserved Noether currents then *have to* include a term coming from the quasi-invariance of the WZ piece: if we wrongly ignored this the algebra of the charges would be

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^\mu)_{\alpha\beta}P_\mu + (C\Gamma_\mu\Gamma_{11})_{\alpha\beta}\hat{Z}^\mu \tag{43}$$

rather than (42). Alternatively, we may find the correct algebra by replacing the quasi-invariant WZ term b by $\tilde{b} = \tilde{b}(x^\mu, \theta^\alpha, \varphi_\mu, \varphi_\alpha, \varphi_{\mu\nu}, \varphi_{\mu\alpha}, \varphi_{\alpha\beta}, \varphi)$, which is *manifestly invariant* since the transformation properties of the additional variables $(\varphi_{\mu\nu}, \varphi_{\mu\alpha}, \varphi_{\alpha\beta}, \varphi)$ remove the quasi-invariance of the WZ term b. Hence, the algebra of charges reproduces (42), and the contributions to $\hat{Z}^{\mu\nu}$ and $\hat{Z}$ are entirely due to the contribution of the additional variables $\varphi_{\mu\nu}$, $\varphi_{\mu\alpha}$, $\varphi_{\alpha\beta}$, φ in the WZ term $\tilde{b}$ (or to the quasi-invariance of $b(x^\mu, \theta^\alpha, \varphi_\mu, \varphi_\alpha)$ if we used b instead).

7 Higher order tensors: the case of the M5-brane

Consider the $D = 11$ M5-brane, which contains a worldvolume two-form field $A(\xi)$. As before, the supersymmetric action is obtained in two steps:

a) First, $H = dA - C$ where C is such that $dC = -(C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\mu\Pi^\nu\Pi^\alpha\Pi^\beta$, and the transformation properties of A are fixed so that H is invariant;

b) Secondly, a WZ term is added to obtain κ-symmetry.

The FDA generated by the LI one-forms Π^α, Π^μ and the three-form H is

$$d\Pi^\alpha = 0 \quad , \quad d\Pi^\mu = (1/2)(C\Gamma^\mu)_{\alpha\beta}\Pi^\alpha\Pi^\beta \quad , \quad dH = (C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\mu\Pi^\nu\Pi^\alpha\Pi^\beta \,. \tag{44}$$

(Note that $ddH \equiv 0$ implies $(C\Gamma^{\mu\nu})_{(\alpha\beta}(C\Gamma_\nu)_{\gamma\delta)} \equiv 0$ which is satisfied for $D = 11$). To find the nontrivial CE $(p+2)$-cocycles for the FDA (44) one may impose the closure condition for h on a general $(p+2)$-form with the correct dimensions. This gives two possible expressions for h. One of them is proportional to $(C\Gamma_{\mu\nu})_{\alpha\beta}\Pi^\mu\Pi^\nu\Pi^\alpha\Pi^\beta = dH$, so it is exact. The other is found to be

$$h \propto (C\Gamma_{\mu_1\ldots\mu_5})_{\alpha\beta}\Pi^{\mu_1}\ldots\Pi^{\mu_5}\Pi^\alpha\Pi^\beta - (15/2)(C\Gamma_{\mu_1\mu_2})_{\alpha\beta}\Pi^{\mu_1}\Pi^{\mu_2}\Pi^\alpha\Pi^\beta H \,, \tag{45}$$

which turns out to be not CE-exact. Hence, there is no solution unless $p = 5$: *the M5-brane, $p = 5$, is characterized by the only non-trivial D=11 (5+2)-CE-cocycle.*

H may be defined as a LI three-form on the extended superspace group of coordinates $\tilde{\Sigma}(\theta^\alpha, x^\mu, \varphi_{\mu\nu}, \varphi_{\mu\alpha}, \varphi_{\alpha\beta})$, namely

$$H = \frac{2}{3}\Pi^\mu\Pi^\nu\Pi_{\mu\nu} + \frac{3}{5}\Pi^\mu\Pi^\alpha\Pi_{\mu\alpha} - \frac{2}{15}\Pi_{\alpha\beta}\Pi^\alpha\Pi^\beta \quad . \tag{46}$$

Moreover, it may be shown that there exists a LI $\tilde{b}$ such that $h = d\tilde{b}$ on a suitably extended superspace [9]. By using the explicit form of the LI one-forms appearing in (46), we may replace the worldvolume two-form $A(\xi)$ by a two-form A on the extended superspace. Also, the gauge transformation $\delta A(\xi) = d\Lambda(\xi)$ is achieved by the one-form $\lambda = \lambda_\mu dx^\mu + \lambda_\alpha d\theta^\alpha$, $\phi^*(\lambda) = \Lambda(\xi)$. Then, defining $\delta\varphi_{\mu\nu}$, $\delta\varphi_{\mu\alpha}$, $\delta\varphi_{\alpha\beta}$ conveniently one obtains $\delta\phi^*(A) = d\Lambda(\xi)$.

The EL equations derived from $I[x^\mu(\xi), \theta^\alpha(\xi), A_{ij}(\xi)]$ are equivalent to the ones corresponding to the new action in which $A(\xi)$ is the pull-back $\phi^*(A)$. In fact, in parallel with the D2 brane case, it is found that $\delta I/\delta\varphi_{\alpha\beta} = 0$ and $\delta I/\delta\varphi_{\mu\alpha} = 0$ are identically satisfied (they are Noether identities) and that only $\delta I/\delta\varphi_{\mu\nu} = 0$ contains a non-trivial part, $\delta I/\delta A_{ij} = 0$. Thus, there remain $\binom{D}{2} - \binom{p+1}{2}$ Noether identities. Consider then $(\phi^*(\varphi_{\mu\nu}dx^\mu dx^\nu))_{ij} = A_{ij}(\xi) = \varphi_{\mu\nu}\partial_i x^\mu\partial_j x^\nu$. Again, the election $x^0 = \tau$, $x^1(\xi) = \xi^1, \ldots, x^5(\xi) = \xi^5$, gives

$$(\phi^* A)_{ij} = \varphi_{\mu\nu}\partial_i x^\mu\partial_j x^\nu = \varphi_{ij}(\xi) + \varphi_{iK}(\xi)\partial_j x^K - \varphi_{jK}\partial_i x^K + \varphi_{KL}\partial_i x^K\partial_j x^L \quad , \tag{47}$$

where $K, L = ((p+1) = 6, \ldots, D-1 = 10)$. The additional degrees of freedom associated with φ_{iK} and φ_{KL} may be removed by suitable gauge transformations. Indeed,

$$\begin{aligned}
\delta_\alpha\varphi_{ij} &= 0 \quad , \quad \delta_\alpha\varphi_{iK} = \frac{1}{2}\alpha_{KL}\partial_i x^L \quad , \quad \delta_\alpha\varphi_{KL} = \alpha_{KL} \quad , \\
\delta_\beta\varphi_{ij} &= -\beta_{iK}\partial_j x^K + \beta_{jK}\partial_i x^K \quad , \quad \delta_\beta\varphi_{iK} = \beta_{iK} \quad , \quad \delta_\beta\varphi_{KL} = 0 \quad ,
\end{aligned} \tag{48}$$

leave $(\phi^* A)_{ij}$ invariant, δ_α removes φ_{KL} (by choosing $\alpha_{KL} = -\varphi_{KL}$), and δ_β sets φ_{iK} equal to zero (for $\beta_{iK} = -\varphi_{iK}$).

The previous discussions of the degrees of freedom for the D2 and M5 worldvolume fields set the pattern for other possible cases.

8 Conclusions

In view of the results described here, it seems natural to conclude that there exists an extended superspace origin for all the worldvolume fields appearing in the various super-p-brane actions: all worldvolume fields may be considered as pull-backs to W for the map $\phi : W \longrightarrow \tilde{\Sigma}$. In other words, *there exists a field/extended superspaces democracy* by which *all superbrane worldvolume fields may be seen as the pullbacks ϕ^* to W of some target extended superspace $\tilde{\Sigma}$ coordinates.*

The appropriate extended superspace $\tilde{\Sigma}$ of the specific theory being considered is determined by an extension of its associated *basic* sTr$_D$ *fermionic group* and, using $\tilde{\Sigma}$, the action of the super-p-brane can be constructed in a manifestly invariant form. In fact, *in this field/extended superspace democracy context, the invariance properties and the non-trivial cocycles of the CE cohomology appear to characterise essentially the different superbranes and their actions* [13] (we might also say that they are *perfect* in the sense of [26]).

Are these extra 'dimensions' *necessary* or just *convenient* for a more geometrical and unified description of superbranes? We already saw that spacetime itself (x^μ) *is* a *consequence* of the *non-triviality* of the D-Minkowski space-valued second cohomology group of the abelian odd translation group sTr$_D$. Thus, it is reasonable to conclude that supersymmetry algebras and superspace groups going beyond the standard ones (see *e.g.* [18, 5, 6, 23, 27, 28, 29, 9, 30, 31]) are *required* for a suitable description of the various superstring and superbrane theories and that, as in the superspace case, Nature makes use of the extension possibilities offered by the non-trivial cohomology groups of sTr$_D$.

Acnowledgements. This work has been partially supported by the DGICYT research grant PB 96-0756 and the Junta de Castilla y León research grant C02/199. The authors wish to thank Igor Bandos and Paul Townsend for helpful discussions.

References

[1] R. Haag, J. T. Lopuszański and M. Sohnius, Nucl. Phys. **B88**, 257–274 (1975).

[2] J. W. van Holten and A. Van Proeyen, J. Phys. **A15**, 3763–3783 (1982).

[3] R. D'Auria and P. Fré, Nucl. Phys. **B201**, 101-140 (1982) (E.: *ibid.* **B206**, 496 (1982)).

[4] P. A. Zizzi, Phys. Lett. **137B**, 57–61 (1984); *ibid.* **149B**, 333–336 (1984).

[5] M. B. Green, Phys. Lett. **B223**, 157–164 (1989).

[6] E. Bergshoeff and E. Sezgin, Phys. Lett. **B354**, 256–263 (1995); see also Phys. Lett. **B232**, 96-103 (1989).

[7] A. Achúcarro, J. M. Evans, P. K. Townsend and D. L. Wiltshire, Phys. Lett. **198B**, 441–446 (1987).

[8] W. Siegel, Phys. Rev. **D50**, 2799–2805 (1994).

[9] E. Sezgin, Phys. Lett. **B392**, 323–331 (1997).

[10] J.A. de Azcárraga and J.M. Izquierdo, *Lie groups, Lie algebras and some applications in physics*, Camb. Univ. Press (1995).

[11] J.A. de Azcárraga and J.M. Izquierdo, *Chevalley-Eilenberg complex and extended objects*, to appear in the *Concise encyclopedia of supersymmetry*, J. Bagger et al. eds., Kluwer, Dordrecht.

[12] J. A. de Azcárraga and P. K. Townsend, Phys. Rev. Lett. **62**, 2579–2512 (1989).

[13] C. Chryssomalakos, J.A. de Azcárraga, J.M. Izquierdo and J.C. Pérez Bueno, Nucl. Phys. **B567**, 293-330 (2000), hep-th/990413.

[14] V. Aldaya and J.A. de Azcárraga, J. Math. Phys. **26**, 1818-1821 (1985).

[15] P. K. Townsend, *p–brane democracy*, in *Particles, strings and cosmology*, J. Bagger, G. Domokos, A. Falk and A. Kovesi-Domokos (eds.), pp. 271–285 (World Sci., 1996), hep-th/9507048.

[16] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo and J. Lukierski, *BPS states in M-theory and twistorial constituents*, to appear in Phys. Rev. Lett., hep-th/0101113.

[17] C. Chryssomalakos, *Stability of Lie superalgebras and branes*, hep-th/0102134.

[18] J. A. de Azcárraga, J. P. Gauntlett, J. M. Izquierdo and P. K. Townsend, Phys. Rev. Lett. **63**, 2443–2446 (1989).

[19] M. Hatsuda and M. Sakaguchi, Nucl. Phys. **B577**, 183-193 (2000).

[20] J. Polchinski, Phys. Rev. Lett. **75**, 4724-4727 (1995).

[21] M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, Nucl. Phys. **B490**, 179-201 (1997); M. Aganagic, C. Popescu and J.H. Schwarz, Nucl. Phys. **B495**, 99-126 (1997); E. Bergshoeff and P.K. Townsend, Nucl. Phys. **B490**, 145-162 (1997).

[22] H. Hammer, Nucl. Phys. **B521**, 503-546 (1998).

[23] M. Sakaguchi, Phys. Rev. **D59**, 046007 (1999).

[24] M. Sakaguchi, JHEP 0004, 019 (2000); see also M. Abe, M. Hatsuda, K. Kamimura and T. Tokunaga, Nucl. Phys. **B553**, 305-316 (1999).

[25] E. Bergshoeff and P.K. Townsend, Nucl. Phys. **B531**, 226-238 (1998).

[26] A. V. Gayduk, V. N. Romanov and A. S. Schwarz, Commun. Math. Phys. **79**, 507–528 (1981).

[27] T. Curtright, Phys. Rev. Lett. **60**, 393-396 (1988).

[28] J. A. de Azcárraga, J. M. Izquierdo and P. K. Townsend, Phys. Lett. **B267**, 366–373 (1991).

[29] I. Bars, Phys. Rev. **D54**, 5202–5210 (1996); see also *Survey of Two-Time Physics*, hep-th/0008164.

[30] A. Deriglazov and A. Galajinsky, Mod. Phys. Lett. **A12**, 1517–1529 (1997).

[31] I. Bars, *S–theory*, Phys. Rev. **D55**, 2373–2381 (1997).